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# PRIME MINITRACK AND BAKER-NUNN ORBITS OF SATELLITE 1959α1 (VANGUARD II)

**by** Hans G. Hertz **God**dard Space Flight Center **G**reenbelt, Md.

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## PRIME MINITRACK AND BAKER-NUNN ORBITS OF SATELLITE 1959 $lpha_1$ (VANGUARD II)

By Hans G. Hertz

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

### ABSTRACT 21100

Concurrent data necessary for making a comparison study of the accuracies of Prime Minitrack and Baker-Nunn observations of Satellite 1959  $\alpha_1$  (Vanguard II) are presented in this report. In all, 244 Prime Minitrack and 187 Baker-Nunn observations are available which were made over a 26-day period while the satellite's transmitter was operating. Prime Minitrack observations were possible only during transmitter operation. Data included here are comprised of Prime Minitrack and Baker-Nunn observations made concurrently.

#### CONTENTS

Abstract	 	 ٠.	٠.			ii
INTRODUCTION	 	 ٠.				1
DISCUSSION OF DATA	 	 				1
CONCLUSION	 	 ٠.				2
ACKNOWLEDGMENTS	 	 				3
References	 	 				3
Appendix A—List of Symbols						41

## PRIME MINITRACK AND BAKER-NUNN ORBITS OF SATELLITE 1959a, (VANGUARD II)

by
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#### INTRODUCTION

Satellite  $1959_{\alpha_1}$  (Vanguard II) was launched 17 February 1959. The transmitter was operating for 26 days through 15 March 1959. Therefore Prime Minitrack observations could be obtained during this period. It was suggested to the author that a comparison of the accuracies of Prime Minitrack and Baker-Nunn observations of a satellite would be interesting. In the present report the data necessary for such a study are presented. The observations used are those made in the period where both types of observations were possible. There were 244 Prime Minitrack and 187 Baker-Nunn observations available which were made during this 26-day period.

#### DISCUSSION OF DATA

For each of the two types, twelve orbits were determined. Their epochs were at 2-day intervals from 19 February 1959 through 13 March 1959 inclusive. For each orbit, the values of the parameters  $S_1$ ,  $S_2$ ,...  $S_6$ ,  $S_{18}$  were determined by differential corrections. Here  $S_1$ ,  $S_2$ ,...  $S_6$  are the values of the constant terms in the expressions for the osculating elements a, e, I,  $\Omega$ ,  $\omega$ , M as given by Brouwer (Reference 1). The quantity  $S_{18}$  is the coefficient of the term  $S_{18}$   $\tau^2$ ,  $\tau$  in units of 100 hours from the epoch, added to Brouwer's expression for the mean anomaly. In each differential correction all observations within 72 hours of the epoch have been used provided they generated residuals,  $\cos \delta \Delta \alpha$  and  $\Delta \delta$ , not larger than 0.10.

The earth parameters used in these solutions are shown in Table 1. The parameters are the equatorial radius R of the earth and the constants k,  $k_2$ ,  $A_{30}$ ,  $k_4$ ,  $A_{50}$ . The last four constants occur in the expression,

$$\begin{split} U &= \frac{\mu}{r} \left[ 1 + \frac{k_2}{r^2} \left( 1 - 3 \sin^2 \beta \right) + \frac{A_{30}}{r^3} \left( -\frac{3}{2} \sin \beta + \frac{5}{2} \sin^3 \beta \right) \right. \\ &+ \frac{k_4}{r^4} \left( 1 - 10 \sin^2 \beta + \frac{35}{3} \sin^4 \beta \right) + \frac{A_{50}}{r^5} \left( \frac{15}{8} \sin \beta - \frac{35}{4} \sin^3 \beta + \frac{63}{8} \sin^5 \beta \right) \right] \end{split}$$

used by Brouwer. The constant k is given by  $\mu = k^2$ .

Several iterations were made. The orbits finally adopted as those based on the Prime Minitrack observations are called briefly Prime Minitrack orbits (PM orbits) and received the numbers 603 through 614. The numbers for the Baker-Nunn orbits (BN orbits) are 631, 616 through 625, and 632. The relationship between these numbers and the epochs is shown in Table 2.

The resulting values for the parameters  $s_1$ ,  $s_2$ , ...  $s_6$ ,  $s_{18}$  are shown in Table 3. There are twelve pairs of lines, one pair each belonging to one of the twelve epochs. The first line in each pair gives the results for the PM orbit, the second for the BN orbit. The orbit numbers given serve to identify the epochs.

Table 4 gives the probable errors for the S<sub>i</sub> obtained. They are arranged in eight pairs of columns. The first column of each pair belongs to PM orbits, the second to BN orbits. One line corresponds to a PM and the corresponding BN orbit for the same epoch. The numbers of this PM and BN orbit are given in the first two columns of the table.

The residuals for the PM observations are shown in Table 5 and those for the BN observations in Table 6. Except for the observations near the beginning and end of the 26-day period every PM or every BN observation appears in three orbits. The PM observations have received serial numbers starting with 1. The numbers of the BN observations are those assigned by the Smithsonian Astrophysical Observatory, without the designation of the year. The observations being precision-reduced observations, all numbers begin with a '7'.

Table 7 gives information as to the accuracy of the representation of the observations by the adopted parameters. The weights given are those found in an iterative process in such a way that they are consistent with the probable errors computed from the residuals. This table shows that the rejection limit of 0.10 referred to on page 1 was too large. Better results would be achieved if it were lowered or if the rejection limit were made dependent on the distribution of errors.

By making additional runs the condition that no observations with residuals of more than 3 times the probable error be included was approximately met.

Table 8 shows the differences  $\Delta S_i$  of the values of the  $S_i$  obtained for a PM orbit and the corresponding BN orbit belonging to the same epoch. The differences are given in the sense PM-BN.

The  $S_i$  are plotted versus the time in Figures 1a-g for the Prime Minitrack orbits and in Figures 2a-g for the Baker-Nunn orbits. If no drag or other non-gravitational forces were present  $S_1$ ,  $S_2$ ,  $S_3$  would be constant,  $S_4$ ,  $S_5$ ,  $S_6$  would be linear functions of the time, and  $S_{18}$  would be zero. The probable errors in Table 4 and Figures 1a-f, 2a-f, and the non-vanishing of  $S_{18}$  indicate that there are deviations from gravitational behavior.

#### CONCLUSION

Additional aspects of this problem are of interest and will be investigated if sufficient time and resources are available. For instance, the dependence of the S<sub>i</sub> on the time could be investigated.

An examination of the  $\Delta S_i$  as to significance could be made. It would also be interesting to examine the residuals of the Prime Minitrack observations with respect to the Baker-Nunn orbits and the residuals of the Baker-Nunn observations with respect to the Prime Minitrack orbits. Finally, the data provide information on the relative accuracy of the two types of observations.

#### **ACKNOWLEDGMENTS**

The computations on which the results of this report are based were carried out with a Differential Correction Program System and some additional programs. The original package was based on the satellite theory by H.G.L. Krause (Reference 2) and was programmed by Miss Elise R. Fisher of the Theoretical Division. Gratitude is also expressed to Mr. Cahill of the same division. The IBM Corporation under Dr. K. Deahl was utilized to substitute Brouwer's theory. Additional work was carried out under the supervision of Mr. A. Shapiro of GSFC. I thank Messrs. H. Bremer and R. Bryant of the Theoretical Division, Messers. R. Danek and J. Weld of the Data Systems Division, and others for help and advice received. Acknowledgement is also due the Smithsonian Astrophysical Observatory for providing the observations prior to publication.

#### REFERENCES

- 1. Brouwer, Dirk, "Solution of the Problem of Artificial Satellite Theory Without Drag," Astronomical Journal, Vol. 64, no. 378, 1959.
- 2. Krause, H. G. L., "Die säkularen und periodischen Störungen der Bahn eines künstlichen Erdsatelliten," *Proceedings of the 7th International Astronautical Congress*, (1956) p. 523.

Table 1

Earth Parameters Used in Solutions.

R	6.378165	megameters
k	4118.0870	degrees megameters 3/2 hour 1
k <sub>2</sub>	0.02201451	megameters <sup>2</sup>
A <sub>30</sub>	0.00059678	megameters <sup>3</sup>
k <sub>4</sub>	0.00111709	megameters <sup>4</sup>
A <sub>50</sub>	0.00000000	megameters <sup>5</sup>

Table 2

Prime Minitrack and Baker-Nunn Orbits.

Epoch 0 <sup>h</sup> AT	J.D.	PM Orbit	BN Orbit
1959 February 19	2436618.5	603	631
21	6620.5	604	616
23	6622.5	605	617
25	6624.5	606	618
27	6626.5	607	619
March 1	6628.5	608	620
3	6630.5	609	621
5	6632.5	610	622
7	6634.5	611	623
9	6636.5	612	624
11	6638.5	613	625
13	6640.5	614	632

Table 3. Parameters of the Prime Minitrack and Baker-Nunn Orbits.

S <sub>18</sub>	+0°,300991 +0,300444	+0.291683	+0.291654 +0.300984	+0.342934	+0,392135	+0.346715	+0,313567	+0.329278	+0.311599	+0.318442	+0.347865 +0.333624	+0.366178 +0.356728
$\mathbf{s}_6$	76°,79972	34.92399	353,19875	311.58485	270.16722	228.91773	187.83316	146.89236	106.08772	65.42991	24.92186	344.56841
	76°,79066	34.93060	353,19961	311.61272	270.17781	228.92077	187.82031	146.86993	106.08707	65.47480	24.93162	344.58977
$\mathbf{S}_{5}$	142°47921	153.01337	163.53023	174.06878	184.57046	195.08509	205.59692	216.11123	226.64131	237.17748	247.70289	258.22770
	142•48830	153.00516	163.52863	174.04075	184.56912	195.08849	205.60918	216.13230	226.63302	237.13776	247.69734	258.21165
$S_4$	177°,88713	170.89423	163.89978	156.90739	149,91296	142,92069	135,92960	128.93408	121.93588	114.92644	107.92496	100,92886
	177°,89304	170.89609	163.90085	156.90589	149,91135	142,91888	135,92626	128.93162	121.93894	114.93593	107.93073	100,93520
83	32°872299	32 <b>.</b> 873182	32 <b>.</b> 874733	32 <b>.</b> 874485	32 <b>.</b> 873771	32 <b>.</b> 873218	32 <b>.</b> 872239	32.872083	32.873338	32.875728	32.875879	32.876008
	32°874176	32 <b>.</b> 873129	32 <b>.</b> 872571	32 <b>.</b> 874596	32 <b>.</b> 874683	32 <b>.</b> 874890	32 <b>.</b> 875379	32.876015	32.876659	32.877521	32.879222	32.879606
$\mathbf{S}_2$	0.16576178	0.16577543	0.16576787	0.16574035	0.16572010	0.16569527	0.16570116	0.16568639	0.16566514	0.16565315	0.16567097	0.16567475
	0.16584522	0.16577952	0.16577466	0.16576405	0.16573206	0.16573413	0.16572163	0.16571999	0.16575409	0.16572557	0.16575377	0.16576193
S <sub>1</sub>	8,3221388	8,3220465	8,3219558	8.3218575	8,3217416	8,3216274	8.3215251	8.3214260	8,3213280	8.3212291	8.3211282	8,3210186
Megameters	8,3221335	8,3220482	8,3219551	8.3218515	8,3217394	8,3216305	8.3215263	8.3214295	8,3213248	8.3212210	8.3211260	8,3210180
oit	603	604	605	606	607	608	609	610	611	612	613	614
	631	616	617	618	619	620	621	622	623	624	625	632
Orbit	PM	PM	PM	PM	PM	PM	PM	PM	PM	PM	PM	PM
	BN	BN	BN	BN	BN	BN	BN	BN	BN	BN	BN	BN

Table 4

Probable Errors of the Parameters of the Prime Minitrack and Baker-Nunn Orbits.

	91	27677 1156	456	1962	3220	1940	202	5687	9095	4643	2321	2313
	$\mathbf{s}_{18} \cdot 10$	10939 3820	2622	4632	3538	3272	3796	4338	3116	1945	2285	2190
	105	714 108	26	315	251	276	366	711	522	707	245	313
	. 9g	669 282	276	260	227	293	203	240	334	190	189	139
	$10^{5}$	576 123	113	234	200	230	322	619	561	707	226	270
<b>J</b> c	. S2 ·	779 367	344	326	311	312	584	314	408	332	341	291
le Error	$10^{5}$	25	81	88	49	70	66	170	161	202	64	59
Probat	. \$4	322 195	152	123	192	172	197	180	270	233	237	509
	106	33 152	194	219	142	127	163	191	131	309	124	207
	. <sub>S</sub> 3	1911 787	625	467	886	758	788	757	1094	713	545	583
	108	3900 167	130	382	312	354	441	1210	1531	711	217	309
	. S2	839 564	551	717	601	785	245	634	567	614	756	589
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BN	rbit	631	617	618	619	620	621	622	623	424	625	632
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Table 5	Residuals of Prime Minitrack Observations.	809	cos 8 ∆a (unit 0°0001)																																			
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Table 7. Accuracy of Representation of Observations

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	1.45 0.06	1.44	1.18 1.80	0.57	1.51 0.55	Weig 1•29 0•23	Weight in R.A. 1.50	1.33 0.12	1.64 0.03	1.59 0.20	1.38 0.31	1.63 0.19
	0.55 1.94	0.56	0.82 0.20	1.43	0.49 1.45	Weig 0.71 1.77	Weight in Decl. 1 0.50 7 1.68	0.67 1.88	0.36	0•41 1•80	0.62 1.69	0.37
	0.0083	0.0086 0.0010	0.0095	0.0130	Prob: 0.0088 0.0044	able Error 0.0109 0.0073	Probable Error of Observation in R.A 88 0.0109 0.0094 0.44 0.0073 0.0080 0.0	in R.A. 0.0101 0.0118	0.0087	0.0087	0.0109	0.0081
	0.0135	0.0137 0.0022	0.0115 0.0021	0.0083	Probi 0.0154 0.0027	Probable Error 54 0.0147 27 0.0026	of Observation in Decl. 0.0163 0.0 0.0035 0.0	in Decl. 0.0142 0.0030	0.0185 0.0016	0.0172 0.0016	0.0163	0.0170 0.0017
					Table 8.	Differences	Differences of Parameters	ø				
	D. C.	${}^{\Diamond}\mathbf{S_1}$		ΔS <sub>2</sub>	${\vartriangle} \mathbf{S_3}$		PM-BN $\triangle S_4$	${}^{\Diamond}\mathbf{S}_{5}$	<b>V</b>	$\delta \mathbf{S}_{6}$	$^{\wedge S_{18}}$	
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Figure 1a through  $1g-S_i$  versus time for Prime Minitrack Orbits.

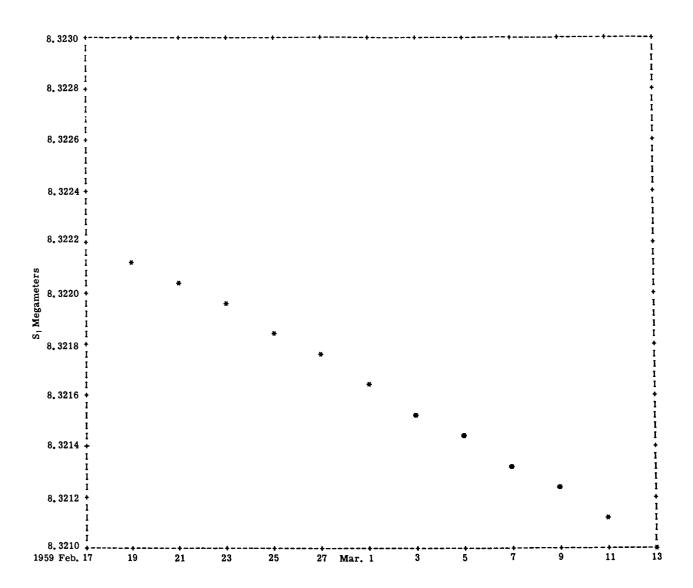


Figure 1a—Values of S<sub>1</sub> (megameters)

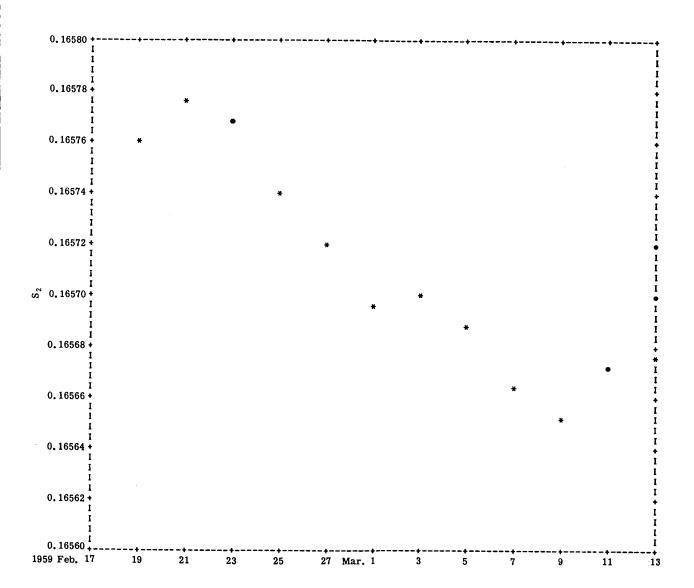


Figure 1b-Values of S<sub>2</sub>

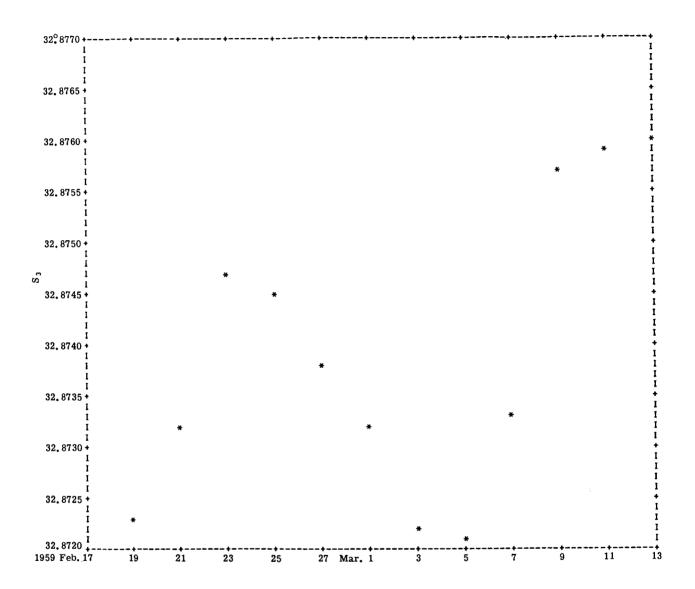


Figure 1c-Values of S<sub>3</sub> (degrees)

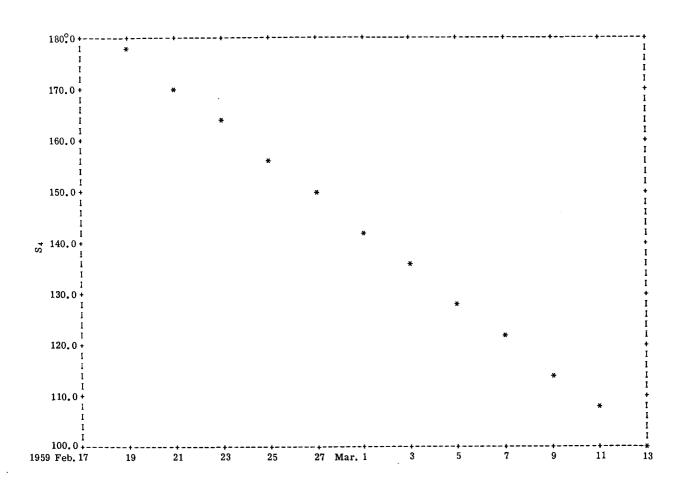


Figure 1d—Values of S<sub>4</sub> (degrees)

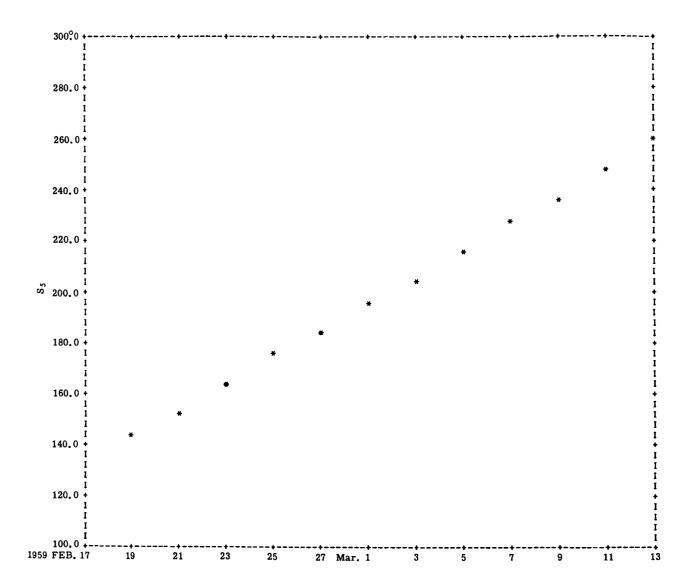


Figure 1e—Values of S<sub>5</sub> (degrees)

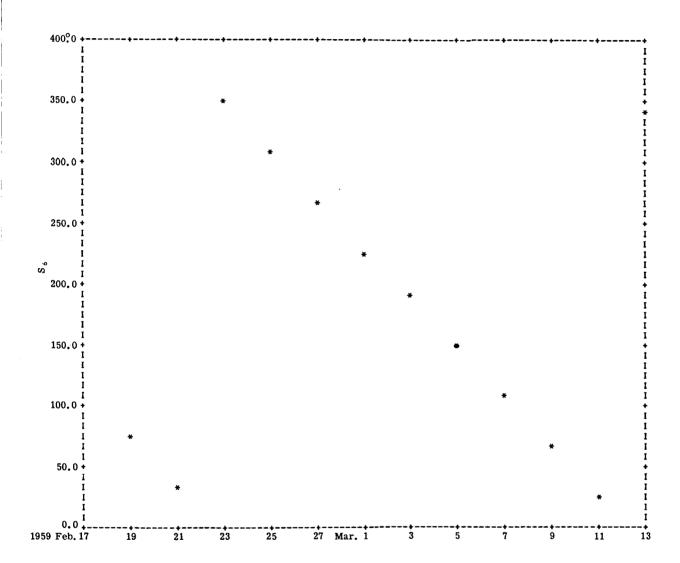


Figure 1f-Values of S<sub>6</sub> (degrees)

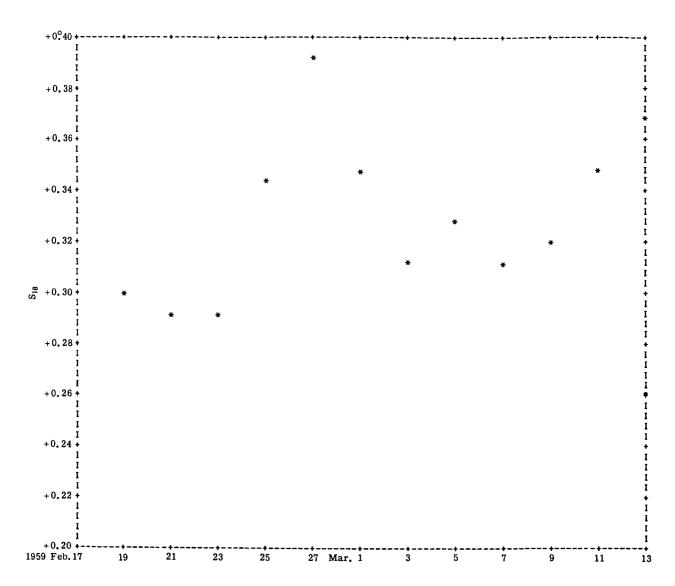


Figure 1g-Values of S<sub>18</sub> (degrees)

Figure 2a through  $2g-S_i$  versus time for Baker-Nunn Orbits.

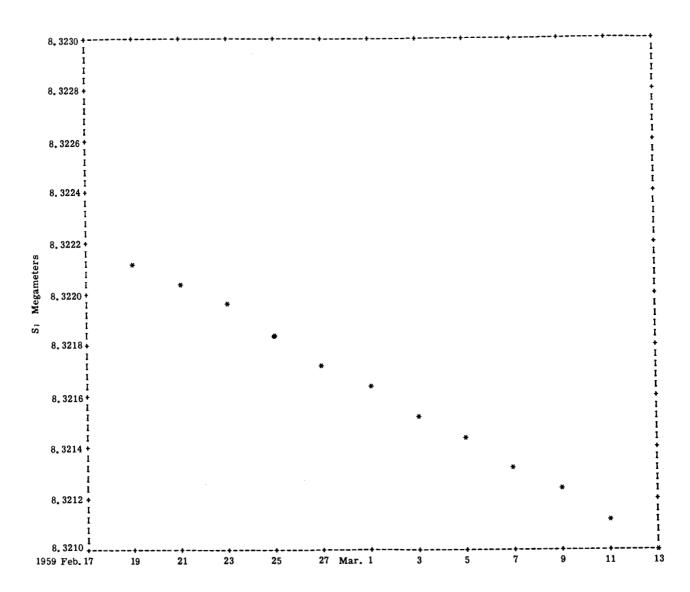


Figure 2a—Values of S<sub>1</sub> (megameters)

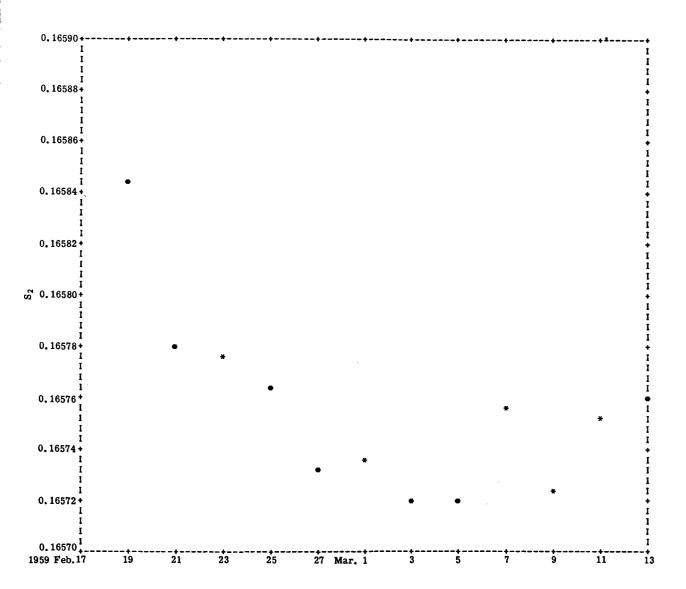


Figure 2b-Values of S<sub>2</sub>

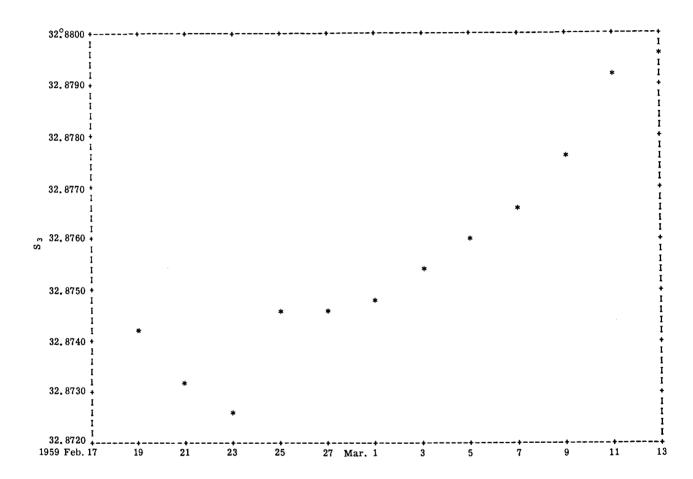


Figure 2c-Values of S<sub>3</sub> (degrees)

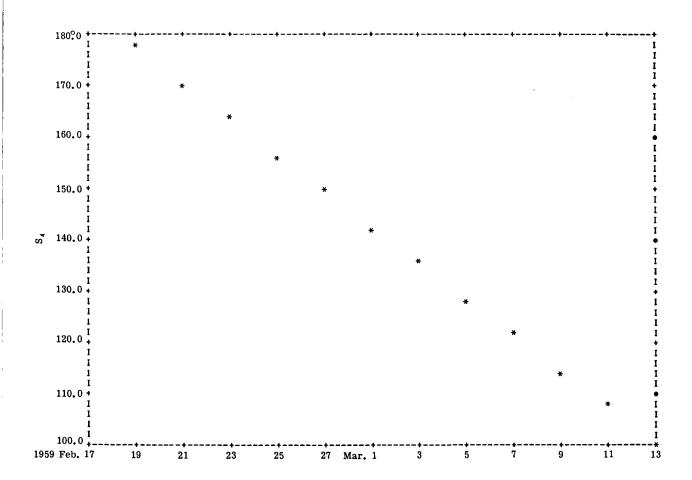


Figure 2d—Values of S<sub>4</sub> (degrees)

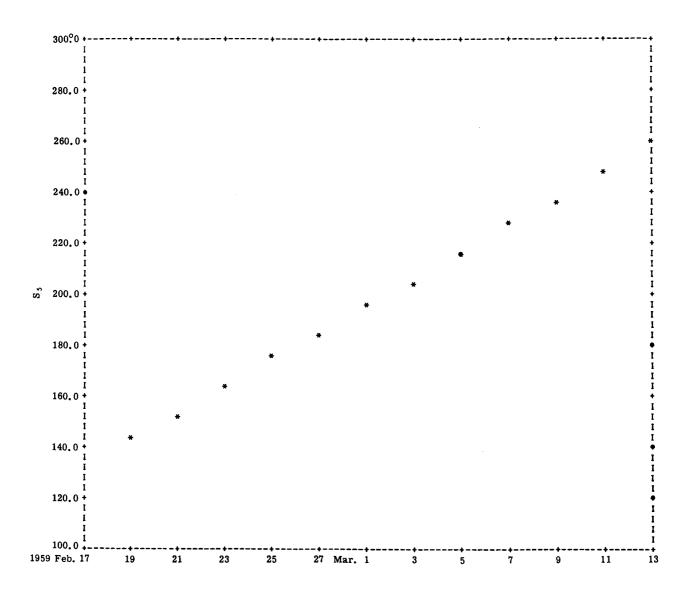


Figure 2e—Values of S<sub>5</sub> (degrees)

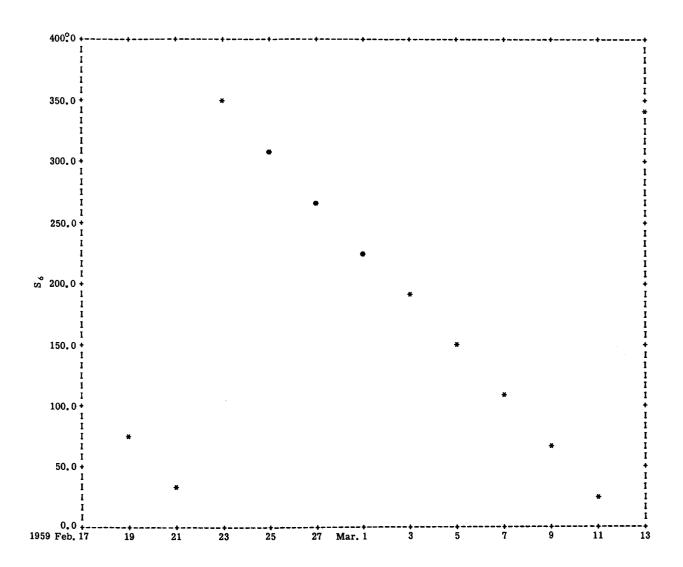


Figure 2f—Values of S<sub>6</sub> (degrees)

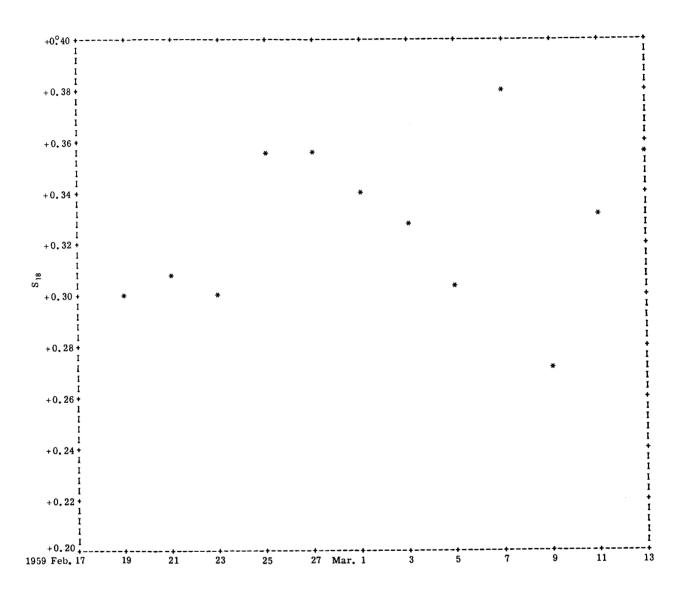


Figure 2g-Values of  $S_{18}$  (degrees)

#### Appendix A

#### List of Symbols

Symbol	Meaning
A <sub>30</sub>	Constant for earth's potential (megameters <sup>3</sup> )
A <sub>50</sub>	Constant for earth's potential (megameters <sup>5</sup> )
I	Inclination of orbital plane to equator (degrees)
M	Mean anomaly
R	Earth equatorial radius (megameters)
a	Semimajor axis (megameters)
e	Eccentricity (non-dimensional)
k	Gravitational constant (degrees megameters 3/2 hour -1)
$\mathbf{k_2}$	Constant for earth's potential (megameters <sup>2</sup> )
k <sub>4</sub>	Constant for earth's potential (megameters 4)
Δδ	Residual in declination
Ω	Longitude of ascending node
α	Right ascension
β	Declination of point for which potential is considered
δ	Declination
$\mu$	k <sup>2</sup> (degrees <sup>2</sup> megameters <sup>3</sup> hours <sup>-2</sup> )
au	Time in units of one-hundred hours
ω .	Argument of perigee
cos δ Δα	Residual in right ascension